Fluidized-bed Heat Transfer: A Generalized Dense-phase Correlation

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A fluidized-bed heat transfer mechanism is proposed that assumes that the chief resistance to heat exchange between a confining wall and a fluidized bed is in the laminar film at the vessel boundary. Heat flow through the film is by conduction. As the solids-particle velocity along the wall will modify film thickness, correlation of film coefficients in terms of particle velocities or the equivalent suggests itself. That heat transport from the boundary of the heating element to the fluidized core proceeds by way of turbulent solids mixing seems indicated by the considerable effect that the solids heat capacity appears to exert on over-all coefficients.

The resulting correlation is examined critically in relation to pertinent literature data. The effect of fluidized-bed parameters on heat transfer coefficients is considered, and applicability and limits of the correlation are ascertained.

Fluidization has been investigated intensively in recent years. Aside from a large number of papers dealing with the mechanics of fluidization, considerable contributions have been made in the field of fluidized-bed heat transfer both between gases and solids and bed and wall. This paper will examine the status of the latter problem and define generalizations as far as possible.

The mechanism of heat transfer in empty and packed columns will be briefly reviewed and extended into a possible mechanism for fluidized columns. In an empty conduit leading a turbulent stream there is always a laminar fluid film in immediate contact with the wall. In this film the radial fluid-velocity component is zero. The laminar film is followed by a buffer layer and thereafter by the turbulent fluid core. If heat is transferred from the wall to the main fluid body, the mechanism must be conduction. At the inner edge of the laminar layer the fluid temperature is considerably less than the wall temperature, but it is still greater than the temperature near the conduit axis. At the buffer layer fluid mixing occurs, as evidenced by the eddy movement of the fluid. Hence beyond the buffer layer heat is transferred by convection to the turbulent-fluid core. From the relatively small temperature drop across the buffer layer (as compared with that across the laminar film) it follows that the main resistance to heat flow is in the laminar film, and so any agency that will reduce the thickness of the laminar layer will obviously also increase the over-all rate of heat transfer. For empty-conduit flow it was found that $h \propto k^n$ where n has an approximate value of 0.65.

In packed columns heat transfer coefficients for the same flow rates are considerably higher than in empty columns, probably because the thickness of the inside laminar layer, and therefore its thermal resistance, has been reduced. Thermal conductivity of the packing particles was found relatively unimportant as far as promotion of wall-bed coefficients is concerned. According to Verschoor and Schuit (*20*), who investigated packed-bed heat transfer with a variety of solids and two widely differing gases (air and hydrogen), $h \propto k^{0.74}$.

As originally reported by Baerg et al. (*2*), heat transfer rates in packed columns operating under countergravity flow increase abruptly as the charge begins to fluidize. This would suggest that the thermal resistance of the laminar layer in contact with the wall is considerably lessened, possibly owing to a reduction in film thickness caused by the vertical motion of the particles. Continued bed expansion (as a consequence of increasing gas velocity) will increase the average interstitial distance between particles. Since heat transfer coefficients are nevertheless, observed to increase for this early expansion range, one is justified in concluding that (direct) interparticle heat flow is negligible in comparison with flow through the film.

The conditions outlined so far and to be discussed further are shown in Figure 1, which emphasizes the over-all behavior of fluidized solids as far as heat transfer coefficient (from bed to wall and vice versa) and fluid-mass velocity are concerned. Along *O-mf* is a fixed bed. At *mf*, the point of initial fluidization, the sudden increase mentioned above is observed. As fluid velocity increases along branch *A*, the bed expands and, although for this section a continued increase in heat transfer is still evident, the rate of increase becomes progressively smaller. At least two opposing influences seem to be at play: (1) the motion of the particles along the wall and the resulting improvement in heat transfer due to the "scouring action" and (2) the increasing state of dilution as far as solids particles is concerned. As is to be expected, this situation should lead to a plateau m and a maximum heat transfer coefficient, h_{max}. The order of magnitude of the maximum may occur anywhere between 100 and 200 B.t.u./(hr.)(sq. ft./°F.); the more exact value will depend on the solids and fluid characteristics.

The descending branch *B* pertains primarily to the fluidized dilute phase. For correlation purposes dilute-phase coefficients may perhaps be considered as essentially those of empty tubes modified by particle traffic along the wall. Dilute-phase particle velocities are high, and velocity changes have relatively less effect on heat transfer than have, for instance, solids-concentration changes. As dilute-phase solids-concentration changes of only a few per cent will affect particle-wall traffic appreciably, dilute-phase coefficients should be readily correlated by solids concentration.

THEORETICAL EQUATION

The proposed theory of heat flow in dense-phase fluidized beds depends upon an understanding of the pattern of particle motion and velocities. If hori-

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Table 1. Summary of Literature

Reference	Solids	Voidage range	Absolute density lb./cu. ft.	Particle-size range, ft.	Type of apparatus and operation	Vessel diameter, in.	Height of heat transfer area, in.	Bed height, in.	Fluids	Flow range, lb./(sq. ft.)(hr.)	T, °F.
(1)	Sands, graphite, soft brick	Dense phase	83–166	8–14 mesh to 36–72 mesh	Steam-jacketed column	1.5	14.5	——	Air	150–1200	
(6)	Aerocat, coke, iron powder	52–69	121–466	0.000252 to 0.000560	Wall steam heating	2.06 and 3.07	23 and 26.5	2–13	Air	68–251	
(8, 9)	Sand, iron catalyst, silica gel	35–75	80–500	0.000129 to 0.00149	Wall steam heat	2.0 and 4.0	25 and 26	12–25	Air CO_2 He, N_2	1.47–1,095	258–413
(13)	Glass beads catalysts, coal	41.7–86.2	63.6–180	0.000250 to 0.0142	Wall electric heating	4.0	3 sections 2, 5, and 2 in.	10–30	Air	79–4,350	
(17)	Glass beads	Dense phase	167–179	0.000179 to 0.00278	Wall water cooling	4.73	7 sections, each 5 in. high	13.2–24.6	Air	23.7–1,542	
(19)	Carborundum, iron oxide, coke, lead fly ash, alloy	Dense phase	112–694	0.000262 to 0.00213	Wall water cooling	3.4	4	16	Air, CH_4, Air, CO_2, Town gas, H_2, N_2	44–779	Approx. 10°–30°C.
(12)	Coal	Dense phase	——	0.000432 to 0.00386	Wall cooling (air)	4.0	24	——	Air	50–1,100	
(2)	Iron powder, sands, glass beads catalyst	38.8–75	119–434	0.000198 to 0.00288	Central electric heat	1.25 in 5.5	4	10	Air	1.85–605	23.5–65.0
(5)	Sand, aluminum graphite, copper catalyst	Dense phase	24.6–27.2	0.00079 to 0.0126	Central cooling	2.31	Immersed cooling coil	——	Air	40–100	87–145
(16)	Silicon carbide, Al-oxide, silica gel	Dense phase	70–243	0.000287 to 0.000817	Center wall cooling	2.0	22	——	Air He CO_2	6.4–101	120–414
(21, 22)	Sand, iron ore	Dense phase	165–330	0.000766 to 0.00197	Internal cooling	1.35 in 22.2	——	47 and 68	Air		
(23)	Glass beads, sand, Socony beads, lead shot	General orienting experiment, dense phase	——	0.002 to 0.0174	Suspended heater	3.94	——	——	Air and water	——	120–229
(3)	Sand, aluminum, calcium-carbonate	54–95	160–167	0.000277 to 0.000822	Wall electric heat	4.0	30	30	Air	96–935	
(4)	Glass beads	Dilute phase	——	0.00023 to 0.0036	Electric heat from outside	1.959	12	——	Air	95–3,780	
(14)	Glass beads	Dilute phase	——	0.000133 to 0.00149	Internal and external heating	2.875 and 1.00	——	——	Air		

zontal particle velocity components are negligible as compared with vertical components, the energy exchange occurring between the fluid and moving particles in the bed may be expressed by (10)

$$u_f A \eta \Delta p = \beta u_p w \qquad (1)$$

Since for solid-gas fluidized systems $A\Delta p \approx w$, Equation (1) becomes

$$u_f \eta = \beta u_p \qquad (2)$$

Vertical particle velocity along the wall is given by $u_p = (L_{mf}R)/\theta$, and substituting into (2) will yield $u_f \eta = \beta(L_{mf}R)/\theta$ where β is the interparticle friction factor. Particle velocity is then related to fluid velocity and bed height by

$$u_p = \frac{u_f \eta}{\beta} = \frac{L_{mf}R}{\theta} \qquad (3)$$

If the particle velocity along the wall will affect film thickness (and thereby heat transfer) the kinematic viscosity of the fluid must be involved. Hence it may be stated that

$$h \propto (f)\left(\frac{1}{z}\right) \qquad (4)$$

where z is the film thickness, which can be expressed by

$$z = K \frac{\mu R}{u_p \rho_F} \qquad (5)$$

Hence for the low velocity range, accompanied by a small degree of bed expansion,

$$h \propto (f)K\left(\frac{u_p \rho_F}{\mu R}\right) = c'(f)\frac{u_f \eta \rho_F}{\beta \mu R}$$

or

$$h = c(f)\left(\frac{G_f \eta}{\mu R}\right) \qquad (6)$$

Besides the usual thermal properties of fluids and solids, constant c should be expected to contain such mechanical quantities as may define the interparticle friction factor (10). The equations contain the fluidization efficiency η, to be discussed in detail later.

DATA

A survey of most literature data pertaining to fluidized-system heat transfer is given in Table 1. The individual correlations are compared in Figure 2. The data pertain to a hypothetical bed composed of a sand 0.006 in. in diameter,

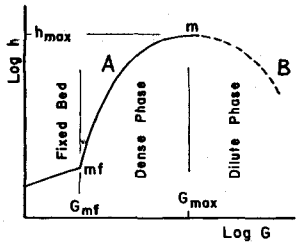

Fig. 1. Heat transfer coefficient in relation to mass velocity for fluidized systems.

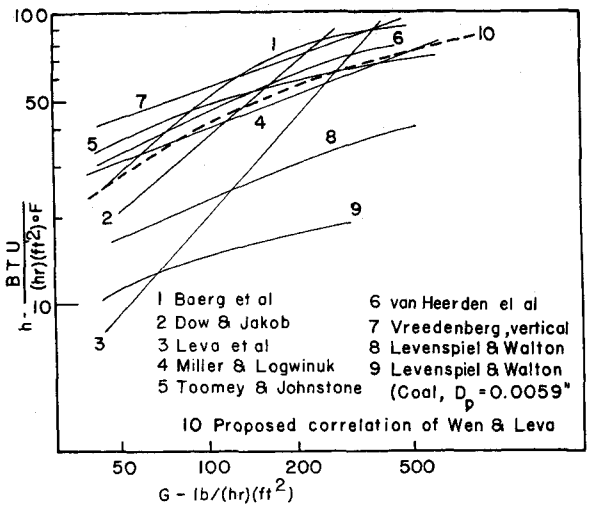

Fig. 2. Heat transfer coefficient in relation to mass velocity for dense phase according to various investigators.

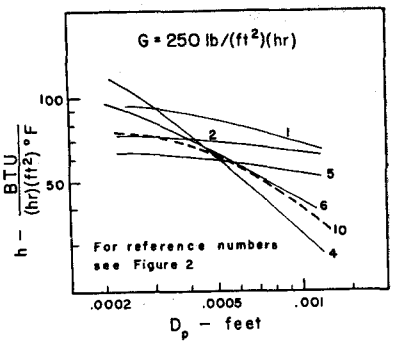

Fig. 3. Heat transfer coefficient in relation to particle diameter at constant mass velocity.

fluidized in air. From Table 1 three broad data classifications are apparent: (1) dense-phase wall-bed heating and cooling, (2) dense-phase center-bed heating and cooling, and (3) dilute-phase heating.

The coefficients reported by Agarwal and Storrow (*1*) are not comparable with the remainder of the dense-phase measurements, because the former were based on a temperature difference that was the arithmetic mean of the terminal differences. Because the arithmetic temperature difference is considerably higher than the true temperature profile in a fluidized heat transfer device (which is usually obtained by local temperature exploration and subsequent graphical integration), the Agarwal and Storrow data appear to be low in comparison.

The early study of Levenspiel and Walton (*12*), reporting on fluidized-coal heat transfer gave unusually low-value date. No ready explanation may be given for this, except perhaps that the method of evaluating the heat flux from the unit might have been in doubt. Since the method was based on a determination of the temperature profile through the layer of insulation around the wall and the thermal conductivity of the layer, uncontrollable effects could readily have interfered with this method of measurement. The later data of Levenspiel and Walton (*13*), pertaining to a considerable variety of solids, are based on a more direct procedure. Heat input was evaluated from powerstat measurements, and temperature explorations were made by a wall-attached and a traveling thermocouple. Nevertheless the coefficients are low compared with the remainder of the data. Admittedly, the writers do not know the reason for the low coefficients, although they strongly suspect that heat losses from that part of the experimental section that did not carry the 9 in. of electrical heating coils could have contributed materially to the low values.

It is interesting in this respect to compare the apparatus of Levenspiel and Walton (*13*) with that of Van Heerden et al. (*19*). In both cases the fluidized bed extended beyond the heat transfer area. In the case of the Van Heerden unit, however, heat losses into the surroundings were negligible, because the bed extending into the unjacketed-tube zone was essentially at room temperature. The major portion of the unjacketed part of the Levenspiel and Walton unit, on the other hand, carried a bed that was roughly about 300°F. above room temperature.

In the light of the now considerable heat transfer data that have been reported for dense-phase fluidized beds, the appreciable deviation of the dense-phase data from the Bureau of Mines reported earlier (*8*, *9*) appears to have been caused by the entrance effect that prevailed with the chosen experimental unit. Careful examination and comparison with techniques of others seem to indicate that the entrance effect is more important than was originally realized. In a fluidized bed of the type considered, the vertical temperature gradient in the first inch or two from the bottom is very great. Although this gradient will not greatly affect an integrated temperature difference if the bed is high (that is, at least a foot or more), a considerable error in the calculated sensible heat quantity will result if the temperature rise of the passing fluid is used. In such a case it is by far better to determine the sensible heat quantity by an over-all energy balance. In the absence of such data the heat transfer coefficients were corrected by recalculating the sensible heat quantity and using the data of Figure 5 of the original article (*8*). Thus if t_x is the exit air temperature and t_i and t_0 are inlet air temperatures for the fluidized column and the empty tube respectively, corrected coefficients result when the originally reported values are multiplied by the ratio $(t_x - t_0)/(t_x - t_i)$. Strictly speaking, a readjustment in temperature difference would also be required. This latter effect, however, is so small as to be well within the limits of the experimental error; therefore the above-mentioned correction will suffice. As examination of the final correlation will reveal, the correction brings the U. S. Bureau of Mines data into satisfactory agreement with other measurements pertaining to heat flow through exterior walls.

The measurements of Campbell and Rumford (*5*) pertained to a relatively wide range of particle diameter and choice of materials; however the mass velocity range was rather limited. Since the construction of the heat transfer unit was such (a helical cooling coil immersed in a cylindrical vessel) that the heat transfer element must have seriously interfered with the quality of fluidization, it was apparent that the Campbell and Rumford data were not of immediate value to the present study, which has as its object the description of broad general trends.

An enlightening effect of apparatus construction may be observed from the data of Vreedenberg (*21*, *22*). He used a wide bed about 22 in. in diameter, into which was immersed a long cylindrical water-cooling element, which could be moved into various positions in the fluidizing charge. His observations are given below:

Cooling-element position	Heat transer coefficient relative to central vertical position
Central, vertical	1.00
4-in. from center, vertical	1.76
8 in. from center, vertical	1.57
Horizontal	1.19

It appears then that in the large-diameter charge the heat transfer coeffi

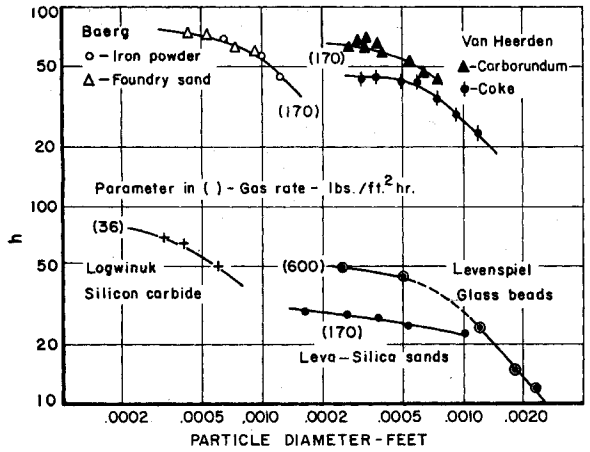

Fig. 4. Heat transfer coefficient vs. particle diameter for various types of solids.

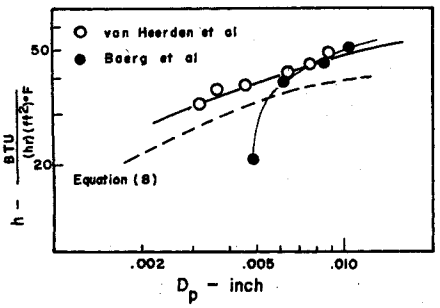

Fig. 5. Heat transfer coefficient in relation to particle diameter at constant reduced mass velocity; G_f/G_{mf} = 3.0 to 3.5.

determining local heat transfer coefficients. Regarding absolute values, the relatively large-scale data of Vreedenberg represent the highest coefficients reported in the literature.

CORRELATION

From Figure 2 it appears that good agreement exists among the measurements of Baerg et. al. (*2*), Dow and Jakob (*6*), Miller and Logwinuk (*16*), Toomey and Johnstone (*17*), and Van Heerden et al. (*19*). In view of the fact that the Baerg and Van Heerden data

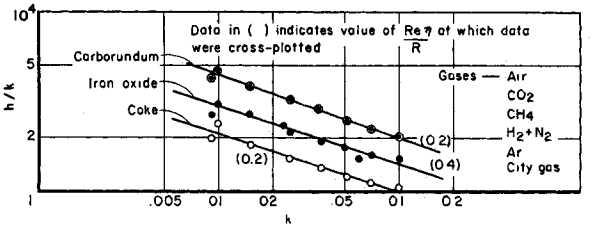

Fig. 6. Effect of fluid thermal conductivity on heat transfer.

cient was lower in the center than in any other position. This is in direct contradiction to all observations of others who investigated central and peripheral heat transfer in equipment of relatively small diameter. The discrepancy is of course due to the fact that in the large charge particle velocities and flow patterns are quite different from those in a small-diameter bed. Whereas in a bed of small diameter (say up to 8 in.) the particles move predominantly upward in the center of the vessel and less vigorously downward near the wall, this type of motion is not likely to prevail in a large-diameter charge, where the pattern of flow is of necessity much less well coordinated. If heat transfer rates and particle movement are ever related explicitly with each other, the observations of Vreedenberg would suggest the interesting possibility of charting internal particle paths and patterns by simply seemed most complete, they were used as the basis for attempting a correlation in line with the suggestion of Equation (6). By the usual methods of plotting and cross plotting, the following relation was obtained:

$$\frac{hD_p}{k} = A\frac{\rho_s^{0.26} c_s^{0.55} D_p^{0.64}}{k^{0.33}}\left(\frac{Re\ \eta}{R}\right)^{0.36} \quad (7)$$

If particle diameter is canceled, it takes the form

$$h = Ak^{0.67}\rho_s^{0.26} c_s^{0.55}\left(\frac{G_f \eta}{\mu R}\right)^{0.36} \quad (8)$$

which is in agreement with Equation (6).

Constant A will vary according to the data level. It will be higher for the data of Baerg than for those of Van Heerden, for instance. The absolute value of A is probably dependent on apparatus construction and experimental techniques. On the basis of an average value of A = 45 the equation reads

$$h = 45k^{0.67}\rho_s^{0.26} c_s^{0.55}\left(\frac{G_f \eta}{\mu R}\right)^{0.36} \quad (9)$$

and the course of the proposed equation indicated in Figure 2 seems well in line with the majority of the data.

Though absolute values predicted by any correlation are important, the general course and trend followed by a correlation of the type discussed is perhaps of even more significance. From Figure 1 it is apparent that the relationship between h and G in a fluidized dense-phase bed can only be approximated by the form $h \propto G^m$. The correlations of Miller and Logwinuk (*16*) and Dow and Jakob (*6*) define heat transfer coefficients in this way. In the case of Miller and Logwinuk the exponent on the mass velocity is about 0.32, whereas for Dow and Jakob it is equal to 0.80. Since the true course of any h vs. G relationship is not exponential, the two correlations will not permit any extension beyond their tested range. The reason why the exponents on the mass velocity differ so greatly for the two sets of data is probably associated with the region of field which the correlations cover. Thus if reference is made again to Figure 1 it appears that Miller and Logwinuk explored the phenomenon nearer the plateau of the curve than did Dow and Jakob. If so, the general level of the Miller and Logwinuk data should be significantly higher than that of the Dow and Jakob data. This is in fact indicated by their measurements.

The correlations of Van Heerden et al. (*19*) and Toomey and Johnstone (*17*) are not of the type just discussed. Instead of presenting heat transfer coefficients as power functions of mass velocity, they make use, in one form or another, of the so-called "reduced" mass velocity (the reduced-mass-velocity concept was also successfully used by Vreedenberg for presenting his large-scale data). As has been pointed out earlier (*10*), use of this concept appears to make allowance for the effect of particle motion on the thickness of the fluid film inside the tube wall and hence should be useful as a correlation factor for fluidized-bed heat transfer data. In effect these correlations make reference to a static bed and by resorting to the use of the reduced mass velocity they introduce a concept similar to the stirring factor suggested recently by Mickley and Fairbanks (*15*).

The correlation proposed in this paper presents the data in their true perspective. As may be seen from Figure 2, the h vs. G path is a curve, rather than a straight line. The curvature is brought about by the inclusion of the product of mass velocity and fluidization efficiency, in the numerator, on one hand and the

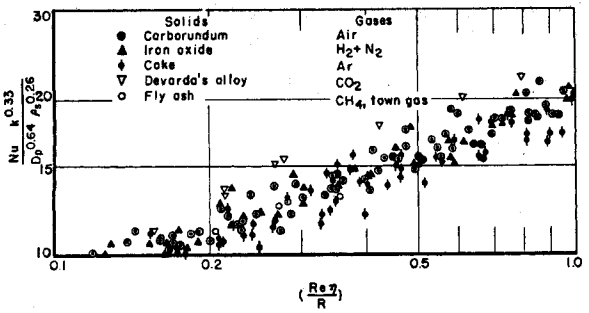

Fig. 7. Details of data of Van Heerden et al.

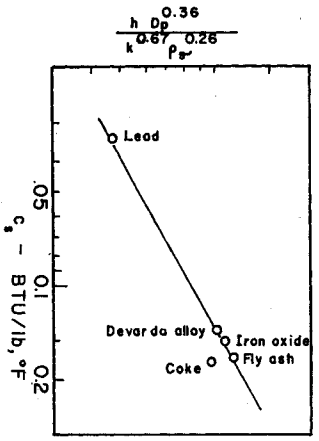

Fig. 8. Effect of solids heat capacity on heat transfer coefficient (data of Van Heerden et al).

bed-expansion ratio (a measure of bed density), in the denominator, on the other.

Another comparison of the correlations is given in Figure 3. According to the various correlations, coefficients were evaluated for a range of particle diameters, with mass velocity kept constant at 250 lb./(sq. ft.)(hr.). The data pertain to the system silica sand–air. Of immediate interest is of course the fact that in all cases the coefficient is inversely proportional to some function of particle size. The fact that for some correlations the dependence seems to be more pronounced than for others finds explanation in the inherent validity range as well as nature of the correlations. A similar plot, but based on the parent data, is indicated in Figure 4. Slopes and data trend are in better agreement than for the calculated values of Figure 3. As far as comparison of the new correlation with the remainder is concerned, the agreement with the Van Heerden, Baerg, and Toomey and Johnstone data seems quite satisfactory. The greatest deviation is indicated by those correlations that present the coefficients as a power function of mass velocity.

Another way of examining the new correlation is to evaluate the heat transfer coefficients for a range of particle diameters, not as before at constant mass velocity, but rather at constant reduced mass velocity. In Figure 5 the course of the new correlation is compared with actually reported measurements. The calculated data, pertaining to silica sands fluidized in air, compare well with the experimental measurements made with foundry sand and carborundum fluidized in air. The reduced-mass-velocity value ranged from about 3.0 to 3.5.

EFFECTS OF THERMAL PROPERTIES

Coefficients are proportional to $k^{0.67}$. This result, established satisfactorily by the data of Figure 6, is in very satisfactory agreement with fixed-bed data. By analogy one may therefore conclude that in the fluidized bed a considerable portion of resistance to heat flow is due to the fluid film inside the heat transfer surface. Heat transfer from the inside of the film to the core of the vessel is probably, as suggested by Van Heerden, by way of turbulent mixing, though particle-particle heat flow is probably negligible. This latter assumption seems indicated by the data of Figure 7. For the solids given, thermal conductivity varied approximately thirty-six-fold, sufficient to indicate a trend if demanded by the heat transfer mechanism. The failure to propagate heat into the bed interior by direct particle-particle action is most likely due to the very small mutual solids contact surfaces that must be assumed to prevail in a fluidized bed. This observation however does not preclude a particle-gas-particle heat-flow mechanism, a phenomenon wherein the entire surface area of the fluidized charge will participate. Although particle-gas coefficients reported so far are not unusually high, the rate of heat propagation into the fluidized core (with heating) must nevertheless be assumed to be rapid, because of the large surface area that prevails in the charge. The mechanism of heat propagation between core and wall just discussed seems well supported by the considerable effect which the solids heat capacity exerts on the magnitude of the coefficients. For the solids examined by Van Heerden the solids heat capacity varied fivefold. The Van Heerden data, cross plotted at a value of $(Re\ \eta)/R = 0.3$, are shown in Figure 8. All the data except those pertaining to coke yield a satisfactory correlation. The deviation of the coke data may perhaps be caused by shape or density differences that are not accountable by heat transfer.

EFFECT OF BED HEIGHT

According to Dow and Jakob (*6*) coefficients vary as the 0.65 power of the diameter-height ratio. Van Heerden and coworkers discussed this point in some detail (*18*) and were rather of the opinion that it was not bed height which affected the coefficients but the height of the heat transfer area instead. A set of experiments disclosed that a decrease in heat transfer area height from about 40 to about 1.5 in. caused an increase in the coefficients of approximately 100%. This was attributed to an entrance effect. As fluidized solids enter a heat transfer section, a maximum temperature difference is established between wall and bed. causing maximum rate of heat flow, Extrapolation of the data to zero bed height yields a heat transfer coefficient of approximately the value of the coefficient calculated from the thermal conductivity of the fluid film, an accepted film thickness of about half a particle diameter being used.

This entrance effect discussed by Van Heerden is definitely important, as is, apparently, the height of the bed. Experimental data of at least two varieties have become available to substantiate this. First there are the measurements of Toomey and Johnstone, which show a definite height effect, and then there are the viscosity measurements of Kramers (*7*), made in certain strata in fluidized beds, which permit conclusions as to uniformity of fluidization and particle movement. The apparatus of Toomey and Johnstone was constructed in such a way that local coefficients could be evaluated along the tube wall. According to a frequently prevailing pattern, the heat transfer coefficients were highest in the base of the column and decreased with increasing bed height. Such an observation would qualitatively also be in line with the viscosity measurements of Kramers. His data indicated that the uniformity in the bed was apparently much better in the base than higher. If the particle movement and the character of particle motion influence the coefficients, then bed height should be expected to affect the coefficients because particle velocity and flow pattern, i.e., quality of fluidization, are known to depend on (in addition to other factors) bed height.

An effect of bed height is also accounted for by the equation which has been proposed by Wicke and Fetting (*24*). On the basis of a film theory a theoretical equation of considerable complexity reports coefficients as being inversely proportional to bed height. Another characteristic of the equation is that it suggests an effect of the height of the heat transfer section, H, on the coefficients. According

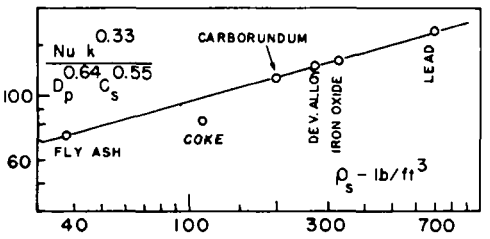

Fig. 9. Effect of solids density on heat transfer coefficients (data of Van Heerden et al.).

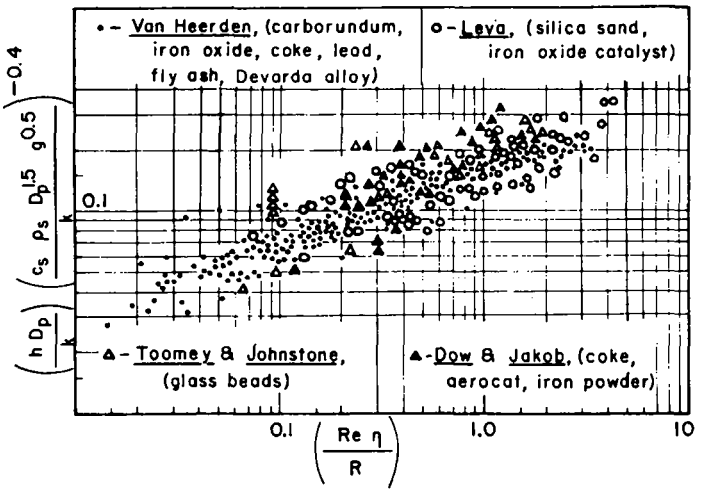

Fig. 10. Final correlation of wall-bed coefficients.

to the form of the correlation and its incorporation of H, one should expect an optimum coefficient for a certain H value. Evaluation of the optimum is complicated, as considerations of other factors, notably film thickness, are involved. The appearance of such an optimum is not indicated by the measurements of Van Heerden (*18*). Optimum bed-height effects have, however, been reported by Toomey and Johnstone (*17*). They are much more readily visualized than are optimum heat transfer surface-height effects.

The effect of solids density upon heat transfer is indicated in Figure 9. The data are those of Van Heerden, with ρ_s varying nearly twentyfold. The correlation is of a quality similar to the already discussed effect of c_s.

DIMENSIONLESS EQUATION

As suggested, one of the objectives of this analysis was to coordinate some earlier observations of solids motion in the bed with heat transfer phenomena through a boundary layer. Although this effort produced an expression that fits the majority of data quite satisfactorily, it is not yet a final correlation. This is clearly evident from its dimensional character. An examination of Equation (7) reveals, however, that dimensional homogeneity would result if the present dimensional group

$$\frac{\rho_s^{0.26} c_s^{0.55} D_p^{0.64}}{k^{0.33}}$$

took the form

$$\left[\frac{\rho_s c_s}{k}\right]^{0.40} D_p^{0.60} g^{0.20},$$

where g is the acceleration due to gravity.

In view of the fact that the foregoing specific exponents are inherently influenced by the method and accuracy of estimating fluidization efficiency and expansion ratio for the solids in question, an over-all exponent of 0.40 may be justified on the basis of the present analysis if the final equation will fit the available data. All available bed-exterior-wall heat transfer data pertaining to the dense phase have been examined in the light of this development and the result is indicated in Figure 10. All data are satisfactorily correlated by the equation

$$\frac{hD_p}{k} = 0.16\left(\frac{c_s \rho_s D_p^{1.5} g^{0.5}}{k}\right)^{0.4} \cdot \left(\frac{G_f D_p \eta}{\mu R}\right)^{0.36} \quad (10)$$

An examination of the equation reveals that it relates the Nusselt group to two other dimensionless groups. The last group is readily recognized as a Reynolds number that includes also the fluidization parameters η and R. The second group includes primarily (with the exception of k) thermal and mechanical properties of the solids phase. These findings are in line with this general development, where the solids velocity along the wall of the vessel was found to be a function of a mechanical constant β, termed the *interparticle friction factor*.

APPLICATION

In order to be able to use the new correlation one must be able to evaluate its components. Evaluation of the physical properties of the fluids and solids poses no special problem. As far as particle diameter is concerned, the geometric mean particle size is suggested. Thus for a narrow-cut solid $D_p = \sqrt{d_1 \times d_2}$, where d_1 and d_2 are adjacent sieve openings. As far as composite sizes are concerned, the equation has not yet been developed sufficiently far to permit suggestion of one or another mode of diameter evaluation.

The fluidization efficiency term in the equation is defined by (*10*)

$$\eta = \frac{G_f - G_e}{G_f}$$

Evaluation of the term has been frequantly discussed. The graphical solution suggested will not only yield η but at the same time will also give a value for R, the bed-expansion ratio. Since the graphical and analytical solutions involve a small amount of labor, nomographic solutions for evaluation of η and R are suggested in Figures 11 and 12. To start with, one computes the value of the reduced mass velocity first. This is obtained by dividing the operating mass velocity by G_{mf}. Values of G_{mf} are conveniently obtained from the nomograph, Figure 13 (*11*). With G_f/G_{mf} known, η is obtained from Figure 11, and by use of η values of R are obtained from Figure 12. Thereafter the equation is readily applied.

Example

A carborundum powder of 0.0038-in. composite diameter arranged in a bed of 4-in. diameter and 10-in. height in the static condition is fluidized by a current of air. For the following conditions and physical properties estimate the heat transfer coefficient through the outside wall.

Air mass velocity is 244 lb./(sq. ft.)(hr.).

Temperature and pressure of the air are such that $\rho_F = 0.075$ lb./cu. ft., $\mu = 0.018$ centipoise, and $k = 0.016$ B.t.u./(hr.)(°F./ft.).

Solids heat capacity c_s is 0.154 B.t.u./(lb.)(°F.)

Solids density $\rho_s = 200$ lb./cu. ft.

Solution

First G_{mf} is estimated. Then it is found that $(\rho_s - \rho_F)\rho_F = (200 - 0.075)\,0.075 = 15$. From the nomograph, Figure 13,

$$G_{mf} = 13 \text{ lb./(sq. ft.)(hr.)}$$

Hence $G_f/G_{mf} = 244/13 = 18.7$ and, from Figure 11, $\eta = 0.82$. From Figure 12, $R = 1.52$. Substitution of these values into Equation (10) yields a value of

$$h = 73 \text{ B.t.u./(hr.)(°F./sq. ft.)}$$

For similar conditions of operation Van Heerden reports $h = 71.5$ B.t.u./(hr.)(°F./sq. ft.) and $G_{mf} = 13.2$ lb./(sq. ft.)(hr.).

It is to be noted that although the conditions of the problem are such that evaluations of η and R by means of Figures 11 and 12 extend into the extrapolated range, the coefficient proposed by the correlation and the procedure are nevertheless in close agreement with experimental evidence.

NOTATION

A = vessel cross section (L^2)
c = constant
D_p = particle diameter (L)

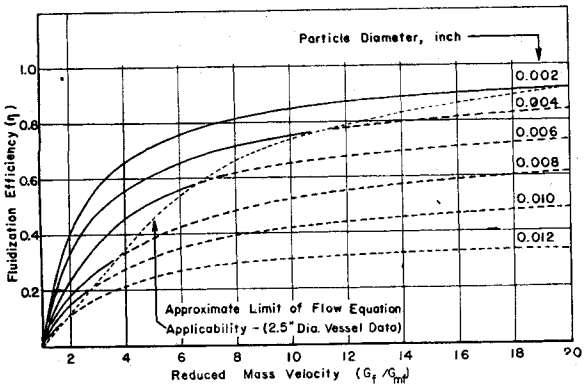

Fig. 11. Fluidization efficiency in relation to reduced mass velocity.

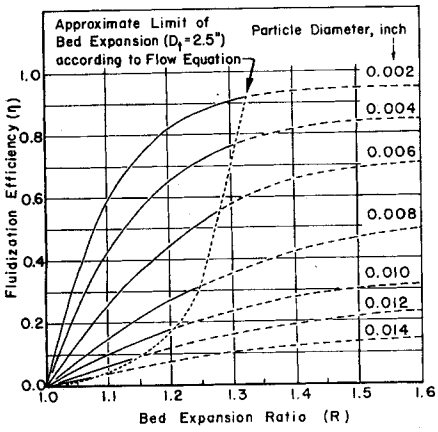

Fig. 12. Fluidization efficiency in relation to bed-expansion ratio.

(f) = function of
g = acceleration due to gravity $(L\theta^{-2})$
G_e = mass velocity for expansion (not fluidization) of a bed $(M\theta^{-1}L^{-2})$
G_f = mass velocity for fluidization of a bed $(M\theta^{-1}L^{-2})$
G_{mf} = mass velocity required to fluidize a bed incipiently $(M\theta^{-1}L^{-2})$
h = coefficient of heat transfer $(H\theta^{-1}L^{-2}T^{-1})$, B.t.u./(hr.) (sq. ft. /°F.), in Equation (7)
k = fluid thermal conductivity $(H\theta^{-1}L^{-1}T^{-1})$ (B.t.u./hr. ft., °F.) in Equation (7)
K = constant
L_{mf} = bed height at onset of fluidization (L)
m,n = exponents (no dimension)
R = expansion ratio of fluidized bed (no dimension)
u_f = superficial fluid velocity $(L\theta^{-1})$
u_p = particle velocity along fluidized bed wall $(L\theta^{-1})$
w = weight of bed (M)
z = film thickness (L)
β = interparticle friction factor (no dimension)
Δp = pressure drop across fluidized bed (FL^{-2})
η = fluidization efficiency (no dimension)
θ = time required for particles to travel from top to bottom of fluidized bed (θ)
μ = fluid viscosity $(ML^{-1}\theta^{-1})$
ρ_F = fluid density (ML^{-3})
ρ_s = solid density (ML^{-3})

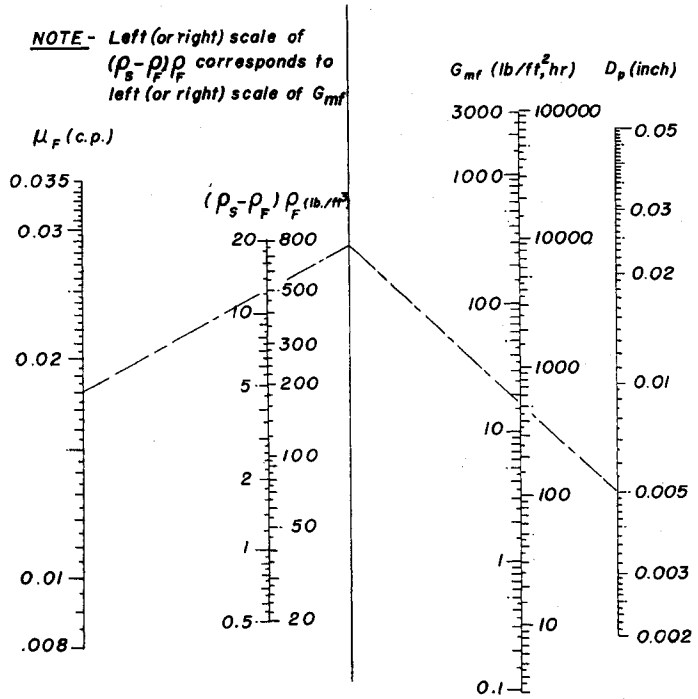

Fig. 13. Nomograph for G_{mf}.

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